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Technical Note

# Buoyant convection of a two-layer liquid system in a cavity: effects of property variations

Angel M. Bethancourt L [a,*], Kunio Kuwahara [b], Jae Min Hyun [c]

[a] Institute of Computational Fluid Dynamics, 1-22-3, Haramachi, Meguro-ku, Tokyo 152-0011, Japan
[b] The Institute of Space and Astronautical Science, 3-1-1, Yoshinodai, Sagamihara, Kanagawa 229, Japan
[c] Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, Yusong-ku, Taejon 305-701, South Korea

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## 1. Introduction

Buoyant convective flow in a square cavity, with the horizontal walls insulated and the two vertical sidewalls maintained at two different constant temperatures $T_c$ and $T_h$, $\Delta T \equiv T_h - T_c$, has served as a benchmark configuration. Validated numerical solutions are available for high Rayleigh numbers (de Vahl Davis [1]).

Recently, studies were made on the convection of two (or possibly more) overlying layers of immiscible liquids in a differentially heated enclosure [2–5]. The early experimental efforts using the water–hexadecane two-layer combination were described by Sparrow et al. [4] and Kimura et al. [5]. In these reports, the shape of the interface between the two liquids was assumed to be horizontal and unperturbed, and the effect of interfacial stress was not taken into account. Companion numerical investigations were also conducted (e.g. [6,7]). A comprehensive numerical solution to the Navier–Stokes equations was secured recently by Bethancourt et al. [8]. It is noted that Ref. [8] included the effect of interfacial stress on a deformable interface. The numerical results of Bethancourt et al. [8] for the case of water–hexadecane system were in accordance with the preceding observations of Sparrow et al. [4] and Kimura et al. [5]. The deformation of the interface under the specific experimental condition in [4] was shown to be very small. This confirmed the reasonableness of the assumptions incorporated in that particular model development of Sparrow et al. [4].

The objective of this note is to supplement the work [8] by providing numerical results over far broader ranges of physical property variations of the two liquids. The solutions in [8] have limited parameter scope and are equipped for verifying the existing experimental findings. The parametric study of this note will disclose the specific and explicit influences of the relative magnitudes of thermophysical properties between the two liquids.

The flow model and geometric layout of the problem are sketched in Fig. 1. The governing Navier–Stokes equations and the associated boundary conditions were stipulated in Ref. [8]. The important element of the present formulation is that the deformation of the interface is to be determined as a part of the solution. The numerical methodologies and attendant solution-procedures were documented in detail in Ref. [8].

## 2. Results

In the ensuing calculations, $H_1/L = H_2/L = 1/2$. Also, for comparison purposes, a standard case (denoted by Run S) was chosen ($Pr_1 = 5.0$, $Gr_1 = 5.26 \times 10^6$, $Ma = 2.9 \times 10^5$, $Ca = 5.4 \times 10^{-4}$), which closely simulates the experiments of [4] using

* Corresponding author.

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**Nomenclature**

| | |
|---|---|
| $Ca$ | Capillary number |
| $Gr$ | Grashof number |
| $h$ | interface height, Fig. 1 |
| $H$ | enclosure height, Fig. 1 |
| $L$ | enclosure length, Fig. 1 |
| $Ma$ | Marangoni number |
| $Nu$ | local Nusselt number |
| $Pr$ | Prandtl number |
| $Ra$ | Rayleigh number |
| $x, y$ | coordinate system, Fig. 1 |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | thermal diffusivity |
| $\beta$ | thermal expansion coefficient |
| $\theta$ | dimensionless temperature |
| $\nu$ | kinematic viscosity |
| $\rho$ | density |
| $\psi$ | stream function |

*Subscripts*

| | |
|---|---|
| 1 | lower layer fluid |
| 2 | upper layer fluid |
| c | cold |
| h | hot |
| $r$ | ratio of the upper layer fluid property to the lower layer fluid property |

water and hexadecane. For Run S, $\rho_r = 0.7731$, $\beta_r = 2.7627$, $\alpha_r = 0.5701$, $\nu_r = 4.4801$.

The stream pattern and temperature field for Run S are displayed in Fig. 2. These general flow characteristics were described in detail in Refs. [4,8].

First, the effect of $\beta_r$, which represents the relative strength of buoyancy in the two layers, is examined by comparing Run 1 ($\beta_r = 0.2761$) and Run 2 ($\beta_r = 27.627$). The ratio of other properties remain unchanged. When $\beta_r$ is small (Run 1), buoyancy is larger in the lower layer and the effect of surface tension (Marangoni effect) becomes more appreciable. The streamlines and isotherms are further concentrated to the sidewall boundary layers in the lower layer. Those produce an intensified counter-clockwise circulation cell in the upper layer-side of the interface. The isotherms are highly distorted in this interface zone. Due to stronger buoyancy, the fluid in the interior in the lower layer is well stratified. For a large $\beta_r$ (Run 2), the intensification of convection in the upper layer is apparent. Furthermore, the deformation of the interface is notable. The counter-clockwise interface cell is weak. The plots in Fig. 3 illustrate the $Nu$ profiles and the interface shape $h(x)$ as $\beta_r$ varies.

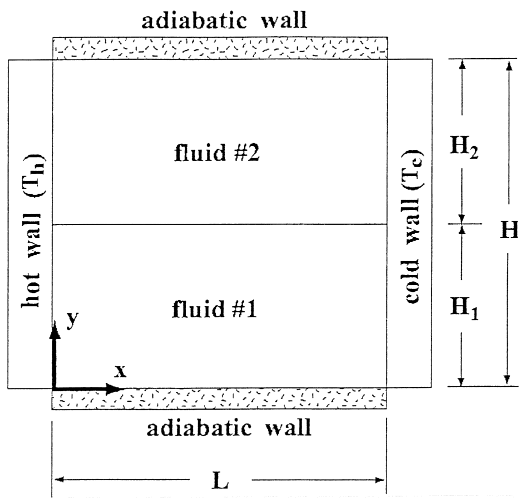


Fig. 1. Schematic of the two-layered system.

When $\beta_r$ is large, the afore-stated intensified convective activity in the upper layer aids the leftward-moving cold flow near the interface. Accordingly, the temperature in the interface zone becomes lower, which leads to a greater distortion of the interface.

When $\alpha_r$ is very small (Run 3) ($\alpha_r = 0.0570$), the overall flow in the upper layer weakens, and a slight amplification of flow is seen in the lower layer. The interior core of the upper layer is mostly at uniform temperature and largely motionless. Also, the boundary layers in the upper layer become thin, and this produces higher values of $Nu$ in the upper layer. The temperature field in the lower layer remains substantially unchanged from that of the standard case. In contrast, when $\alpha_r$ is large (Run 4) ($\alpha_r = 5.70$), fluid motions in the upper (lower) layer are intensified (suppressed), and a weak counter-clockwise cell appears below the interface near the cold sidewall. In the upper layer, the thermal boundary layer is thicker, which causes the $Nu$ values to decrease. Only a slight deformation is seen in the interface shape (note the difference in scales used in the ordinates of Figs. 3 and 4). This suggests that $\beta_r$ is the primary factor in the determination of the distortion of the interface.

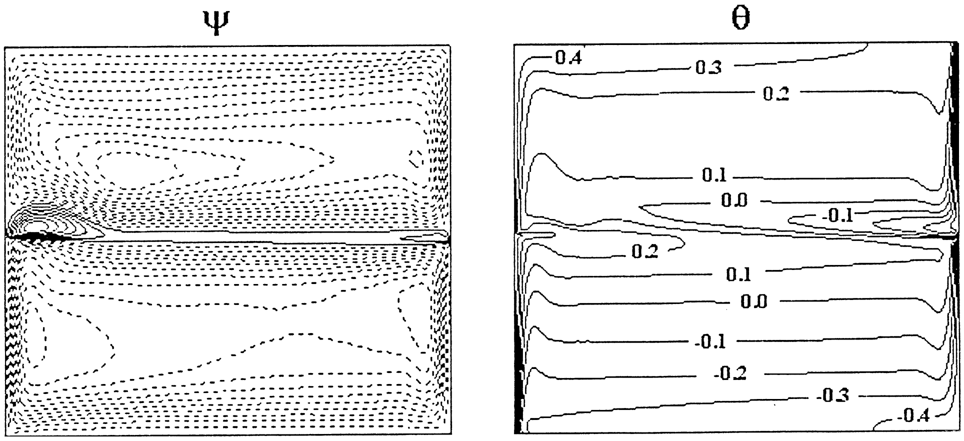


Fig. 2. Run S: $|\psi_1|_{max} = 1.73 \times 10^{-2}$, $|\psi_2|_{max} = 1.55 \times 10^{-2}$ $(Pr_2 = 39.2, Gr_2 = 7.24 \times 10^5)$.

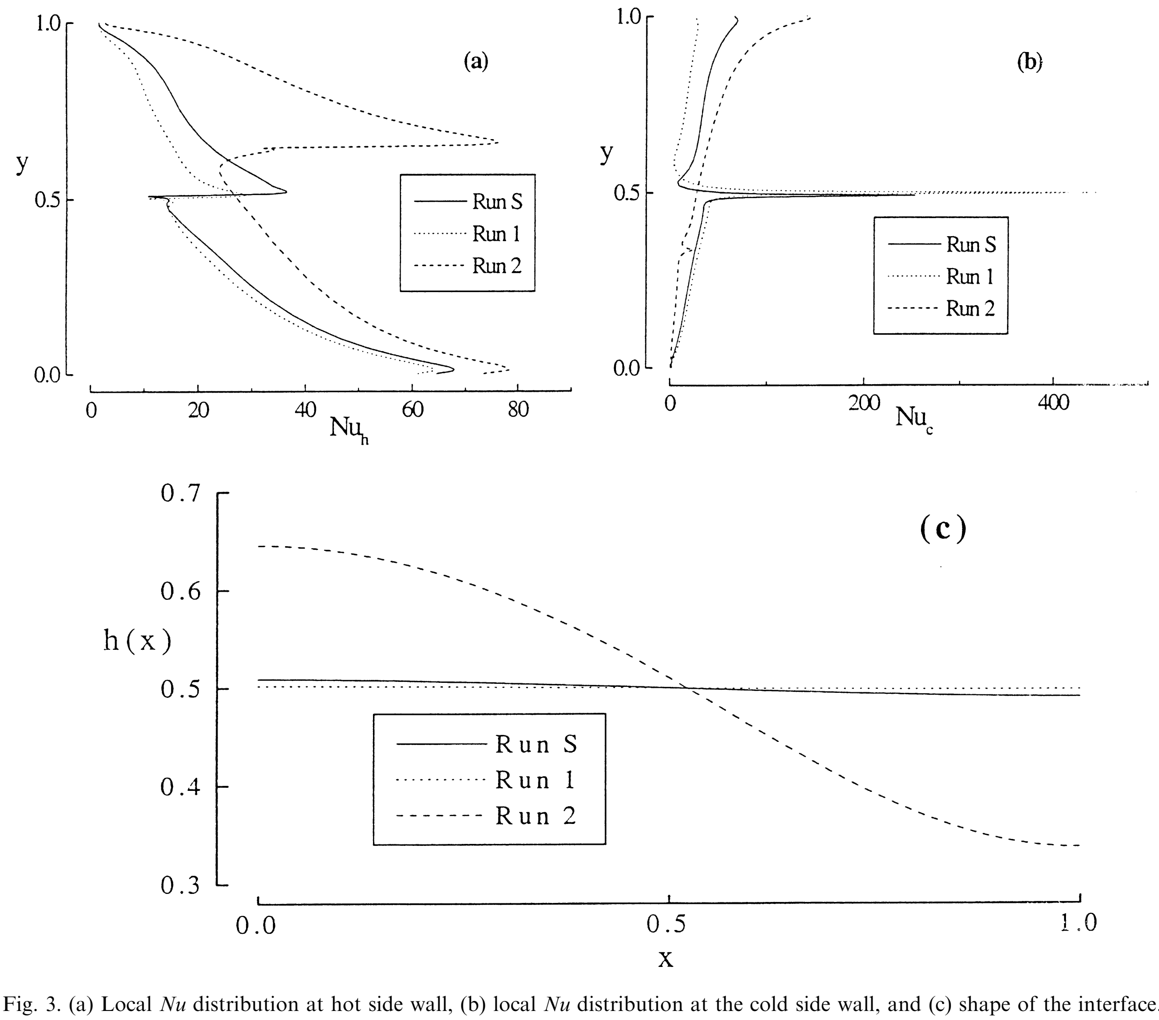


Fig. 3. (a) Local $Nu$ distribution at hot side wall, (b) local $Nu$ distribution at the cold side wall, and (c) shape of the interface.

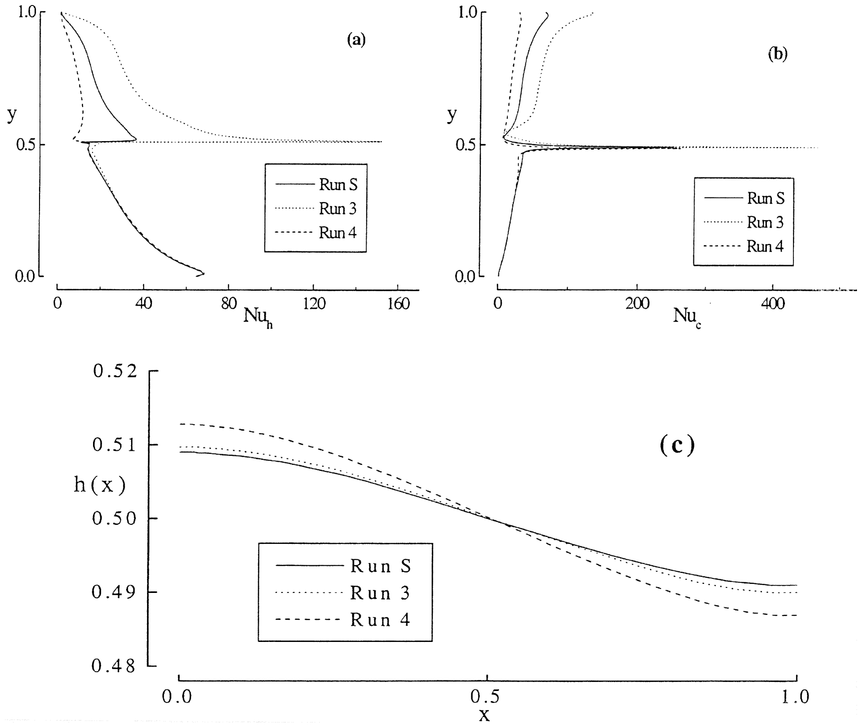


Fig. 4. (a) Local *Nu* distribution at hot side wall, (b) local *Nu* distribution at the cold side wall, and (c) shape of the interface.

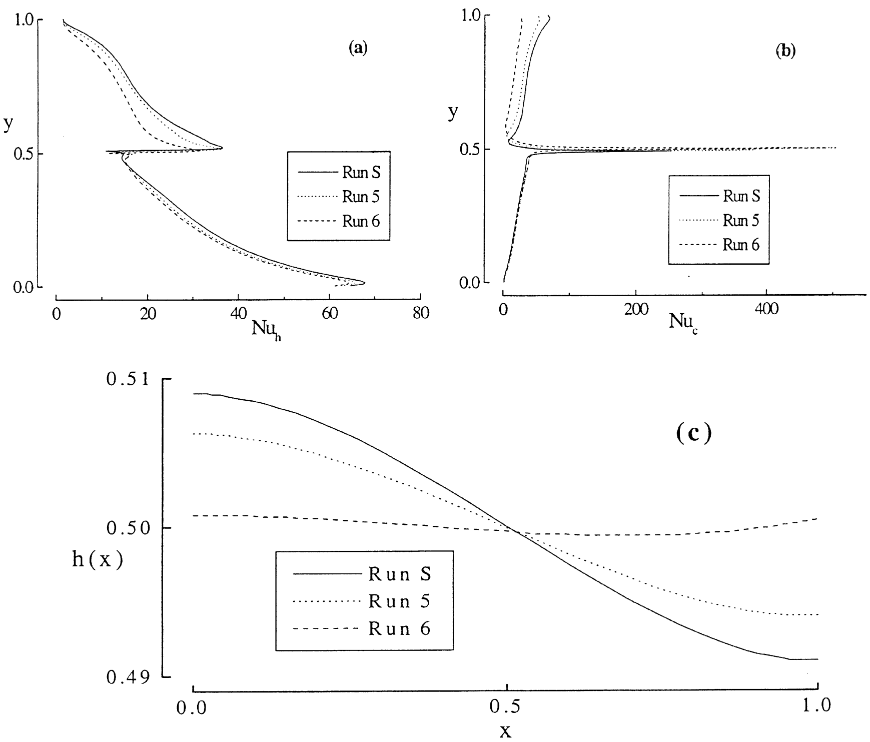


Fig. 5. (a) Local *Nu* distribution at hot side wall, (b) local *Nu* distribution at the cold side wall, and (c) shape of the interface.

The effect of $\nu_r$, the ratio of kinematic viscosities, is illustrated in Fig. 5. Since $\nu_r \equiv \mu_r/\rho_r$, two ways are possible to effectuate $\nu_r$, i.e., by changing $\mu_r$ or $\rho_r$. In Runs 5 and 6, $\mu_r$ is left unchanged, and $\rho_r$ is allowed to vary to result in variations in $\nu_r$.

As $\nu_r$ is increased (Run 5, $\nu_r = 8.96$), minor modifications are seen in the thermal field, and the size of the counter-clockwise cell in the upper layer-side of the interface gets slightly increased, in particular in the vicinity of the hot sidewall. For a large increase in $\nu_r$ (Run 6, $\nu_r = 44.8$), motions in the upper layer weaken, and the counter-clockwise cell grows in strength and size to occupy a substantial portion of the upper layer. It follows that, due to the weakening motions in the upper layer, the *Nu* values in the upper layer are reduced (see Fig. 5). The corresponding changes in the *Nu* values in the lower layer are small, and the deformation of the interface is minimal.

## 3. Conclusions

The flow and heat transfer characteristics pertinent to the standard case (Run S) are consistent with the experimental observations. In comparison to the standard case, variations in the ratios of physical properties between the two layers can cause significant alterations in the flow.

When $\beta_r$ is increased, flows become intensified in the upper layer, and a large deformation of the interface is observed. When $\alpha_r$ is small, the interior core of the upper layer is largely motionless with uniform temperature, and an increase in the *Nu* values is discernible. As $\alpha_r$ increases, flows become vigorous in the upper layer. As $\nu_r$ increases, the *Nu* values in the upper layer decrease.

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